Calculator Section – Problems 1-24

1. Evaluate the following:
   a) \[ \int_{0}^{0.2} \tan^2(x)\,dx = \]
   b) \[ \int_{0}^{0.4} \sec^2(x)\,dx = \]
   c) \[ \int_{2}^{4} \cos(x^3 - 1)\,dx = \]

2. For \(4x^3 - 3xy^2 + y^3 = 28\), find \(\frac{dy}{dx}\) at the point \((3, 4)\) on the curve.

   A. \(\frac{5}{2}\)  
   B. \(\frac{4}{3}\)  
   C. \(\frac{5}{12}\)  
   D. \(-\frac{7}{6}\)  
   E. \(\frac{2}{3}\)

3. Given \(V = \frac{4}{3} \pi r^3\); \(S = 4\pi r^2\) for a sphere, and the fact that the radius of the sphere is decreasing at 1/2 inches per minute, find the rate at which the volume is changing, in cubic inches per minute for a sphere at the time when the radius is 6 inches.

4. from AP 2002 AB 5

   A container has the shape of an open right circular cone, as shown. The height of the container is 10 cm and the diameter of the opening is 10 cm. Water in the container is evaporating so that its depth \(h\) is changing at a constant rate of \(-\frac{3}{10}\) cm/hr. \((V = \frac{1}{3} \pi r^2 h)\)

   a. Find the volume \(V\) of water in the container when \(h = 5\) cm. Indicate units of measure.

   b. Find the rate of change of the volume of water in the container, with respect to time, when \(h = 5\) cm. Indicate units of measure.
5.

Consider the graph shown to be the velocity of a turtle moving left and right in relation to a mailbox, with velocity in feet/minute for t minutes. The turtle, at t = 0 is standing 1 foot to the right of the mailbox.

a. Where is the turtle in relation to the mailbox at t = 4 minutes?

b. Where is the turtle in relation to the mailbox at t = 5 minutes?

6. A 25 ft ladder is sliding down a wall so that the base is moving along the ground at 3 ft/hour. When the ladder is 10 feet high (on the wall), how fast in ft/hour is the top of the ladder moving along the wall?

A. -3 ft/hour B. -6.874 ft/hr C. -21.869 ft/hr D. 21.869 ft/hr E. 6.874 ft/hr

7. Use the table at the right to answer the following.

If \( f \) is a continuous and differentiable function, then approximate the value of \( f'(3) \).

A. 4.5 B. 4 C. 2 D. 1/2 E. 1/4

<table>
<thead>
<tr>
<th>x</th>
<th>f(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
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<td>2</td>
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<td>5</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>6.2</td>
</tr>
</tbody>
</table>

8. Given \( f'(x) = 2\sin^4(2x) - 3\sin^3(4x) \). Over the open interval (0, 0.7), at what value of x is the tangent line to \( f \) horizontal?

A. 0.390 B. 0.555 C. 0.368 D. 0.567 E. 0.195

9. At which x-value over the interval [0, 2] does the graph of \( f \) have a relative minimum? (refer to \( f'(x) \) in #8)

A. 1.938 B. 1.146 C. 0.368 D. 1.571 E. 0.567
10. TRUE or FALSE, given the graph of \( f \) to the right.

a. \( \lim_{x \to 1} f(x) \) exists. ____

b. \( \lim_{x \to -2} f(x) \) exists. ____

c. \( f \) is continuous over the interval \([-1, 4]\). ______

d. \( f \) is differentiable over the interval \([-1, 4]\). ______

e. \( f \) is differentiable over the interval \([-3, -1]\) ______

11. A rectangle with one side on the \( x \)-axis has its upper vertices on the graph of \( y = \cos x \) as shown in the figure below. What is the maximum area of the rectangle?

A. 0.860        B. 0.982        C. 1.122        D. 3.426        E. 6. 577

12. The function \( f \) is continuous and differentiable over the interval \([-10, 10]\). If \( f(-4) = 3 \) and \( f(6) = 23 \) then tell (true or false) which statements must be true.

a. Over \([-4, 6]\), the function \( f \) has a maximum value. ____

b. Over \([-4, 10]\) there exists a value \( c \) so that \( f(c) = 0 \). _____

c. Over \([-4, 10]\) there exists a value \( c \) so that \( f(c) = 10 \). _____

d. Over \([-4, 10]\) there exists a value \( c \) so that \( f'(c) = 0 \). _____

e. Over \([-4, 10]\) there exists a value \( c \) so that \( f'(c) = 2 \). _____

f. Over \([-4, 10]\), \( f \) is increasing.____

g. Over \([-4, 10]\) \( f'(c) \) exists for every \( c \) in \([-4, 10]\). ______

13. What is the slope of the line tangent to \( y = x^2 - \cos^2(3x) \) at the point on the curve when \( x = 0.861 \)?
14. The graph of \( y = x^{3/2} \) is shown above, on the interval \([-5, 5]\). Which is true?

A. The graph is continuous and differentiable over the interval.
B. The graph is continuous but not differentiable over the interval.
C. The graph is not continuous but is differentiable over the interval.
D. The graph is not continuous, nor differentiable over the interval.

15. The graph of a twice-differentiable function \( f \) is shown below.

Which of the following is true?

A. \( f(1) < f'(1) < f''(1) \)
B. \( f(1) < f''(1) < f'(1) \)
C. \( f''(1) < f(1) < f'(1) \)
D. \( f'(1) < f''(1) < f(1) \)
E. \( f''(1) < f'(1) < f(1) \)

16. Given \( f(x) = 2\sin^4(2x) - 3\sin^3(4x) \). Find the interval of x-coordinates for which the graph of \( f \) is concave up on the interval \([0, 0.78]\).
17. Consider the average rate of change of the graph of \( f \) above, between the points shown. Between which two points is the average rate of change the least?

A. Points A and B  
B. Points A and D  
C. Points B and D  
D. Points B and C  
E. Points A and C

18. The graph of \( f \) is shown in the figure above. Which of the following could be a graph of the derivative of \( f \)?

19. A particle moves along the x-axis so that at time \( t \geq 0 \) its position is given by \( x(t) = \cos \sqrt{t} \). What is the velocity of the particle at the first instance the particle is at the origin?

20. Two people are walking away from each other, one heading north at 3 miles per hour and the other heading east at 4 miles per hour. How fast are the two people moving away from each other after walking for two hours?
21. Suppose silver is being extracted from a mine at a rate given by \( A'(t) = 100e^{-0.2t} \), where \( A(t) \) is measured in tons of silver and \( t \) in years from the opening of the mine. Which is an expression for the amount of silver extracted from the mine in the first 5 years of its opening?

A. \( A'(5) \)  
B. \( \int_{0}^{5} A(t)dt \)  
C. \( \int_{0}^{5} A'(t)dt \)  
D. \( A(5) \)  
E. \( A'(5) - A'(0) \)

22. Given \( f'(x) = 2\sin^4(2x) - 3\sin^3(4x) \), at which \( x \)-coordinate(s) below does the graph of \( f(x) \) change concavity over the interval (0, 2)?

A. 1.938  
B. 1.146  
C. 0.667  
D. 1.571  
E. 0.567

23. At which interval is the graph of \( f(x) \) (for \( f'(x) \) defined in #22) concave up over the interval (0, 0.8)?

A. (0, 0.8)  
B. (0.368, 0.785)  
C. (0, 0.368)  
D. (0.368, 0.8)

24. If \( v(t) = 2t^3 - 15t^2 + 24t + 1 \) is the velocity function for a particle moving along the x-axis, for time \( t \) seconds, find each of the following:

a. when the particle changes direction for \( t \geq 0 \).

b. for what intervals of time \( (t \geq 0) \) the particle is moving to the left.

c. for what intervals of time \( (t \geq 0) \) the particle has increasing speed.

d. for what intervals of time \( (t \geq 0) \) the acceleration of the particle is positive.
25. If \( y = \cos^3(4x - 1) \) then \( \frac{dy}{dx} = \)

A. \( 12 \cos^2(4x - 1) \)  
B. \( -12 \sin^2(4x - 1) \)  
C. \( -3 \cos^2(4x - 1) \)  
D. \( -12 \cos^2(4x - 1) \sin(4x - 1) \)  
E. \( -3 \cos^2(4x - 1) \sin(4x - 1) \)

26. Which is an equation of the tangent line at \( x = 2 \), for the graph of \( y = 6x^2 - 12x + 1 \)?

A. \( y = 19x + 1 \)  
B. \( y = -5x + 1 \)  
C. \( y = 12x - 23 \)  
D. \( y = -5x + 7 \)  
E. \( y = 12x + 8 \)

27. Evaluate each of the following limits:

a. \( \lim_{n \to \infty} \frac{5n^2}{2n^2 + 500n} = \)

b. \( \lim_{x \to \infty} \frac{5x^2 - 3x + 9}{1 - x^2} = \)

c. \( \lim_{x \to \infty} \frac{5x + 9}{1 - x^2} = \)

d. \( \lim_{h \to 0} \frac{5(x + h)^4 - 5x^4}{h} = \)

e. \( \lim_{x \to 5} \frac{x^2 - 5x}{5 - x} = \)

f. \( \lim_{x \to 4} \frac{x^2 - x - 12}{x^2 - 4x} = \)

g. \( \lim_{h \to 0} \frac{\sin(\pi + h) - \sin(\pi)}{h} = \)
28. Let \( g(x) = \begin{cases} 
\frac{x^2 - 4}{x - 2}, & x \neq 2 \\
3, & x = 2 
\end{cases} \)

Which of the following statements are TRUE?

I) \( \lim_{x \to 2} g(x) \) exists 
II) \( g(2) \) exists  
III) \( g \) is continuous at \( x = 2 \)

A) Only I  
B) Only II  
C) I and II only  
D) None of them  
E) All of them

29. A function \( f(x) \) equals \( \frac{x^2 - x}{x - 1} \) for all \( x \) except \( x = 1 \). For the function to be continuous at \( x = 1 \), the value of \( f(1) \) must be

A. 0  
B. 1  
C. 2  
D. \( \infty \)  
E. none of these

30. The graph of \( y = \frac{2x^2}{4 - x^2} \) has

A. two horizontal asymptotes  
B. two horizontal and one vertical asymptotes  
C. two vertical but no horizontal asymptotes  
D. one horizontal and one vertical asymptote  
E. one horizontal and two vertical asymptotes

31. Find the inflection POINT(S) of the graph of each of the following:

a. \( y = 2x^3 - 6x^2 \)

b. \( y = 2x^4 - 8x \)
32. Shown to the right: the graph of \( f \) over \([-5, 5] \times [-3,3]\). Find each of the following:

a. \( \lim_{x \to -3} f(x) = \) ____________  
b. \( f(-3) = \) ____________

c. \( \lim_{x \to 0} f(x) = \) ____________  
d. \( f(0) = \) ____________

e. \( \lim_{x \to 2^+} f(x) = \) ____________  
f. \( \lim_{x \to 2^-} f(x) = \) ____________

g. \( \lim_{x \to 4^-} f(x) = \) ____________  
h. \( \lim_{x \to 4^+} f(x) = \) ____________

i. \( \lim_{x \to 4^+} f(x) = \) ____________  
j. \( f(3) = \) ____________

k. Estimate the x-coordinates where \( f(x) = 1 \).

l. \( \lim_{x \to 5} f(x) = \) ____________

33. \( f(1) = 8 \), \( f'(1) = 3 \), \( f''(1) = -4 \) for a twice-differentiable function \( f \). Write an equation of the line tangent to \( f \) at \( x = 1 \).

A. \( x - y = -2 \)
B. \( 3x - y = -5 \)
C. \( 3x + y = 11 \)
D. \( 3x - y = 7 \)
E. \( 3x + y = -1 \)

34. Given \( f(x) = 3x^2 - 2x + 1 \), and \( f(1) = 2 \) , then find \( (f^{-1}(2))' \).

A. 9  
B. \( \frac{1}{10} \)  
C. 4  
D. \( \frac{1}{4} \)  
E. \( \frac{1}{2} \)

35. Consider the implicitly defined function, \( x^2 + xy - y^2 = 1 \).

a) Which of the following point(s) lie on the curve defined by the function?

(2, 3)  
(1, 1)  
(-1, 2)  
(1, 0)

b) Determine the tangent line for each of the above points that lie on the curve.

36. If \( f(x) = (x^2 + 3x + 6)^3 \), then \( f'(0) = \) ?
37. What is \( f'(x) \) if \( f(x) = \frac{x^3}{\sin x} \)?

38. At any time \( t \), the position of a particle, in feet, moving along an axis is \( s(t) = \frac{t^3}{3} - 2t^2 + 3t \). What is the acceleration, in \( \text{ft/ sec}^2 \) of the particle at \( t = 4 \) seconds?

39. Given \( f(x) = e^{5x} \), find \( f''(x) \).

40. If \( g \) is a continuous function on the closed interval \([-4, 4]\) and \( g(-4) = 2 \) and \( g(4) = 7 \), which of the following statements is necessarily TRUE?
   
   A. The absolute maximum value for \( g \) on \([-4, 4]\) is 7
   B. If \(-4 < x < 4\), \( g(x) = 5 \) for at least one value of \( x \)
   C. \( g(x) = 0 \) for at least one value of \( x \) on the closed interval \([-4, 4]\)
   D. \( g \) is increasing for all \( x \) on \([-4, 4]\)
   E. The absolute minimum value of \( g \) on \([-4, 4]\) is 2

41. Give an expression for \( \frac{dy}{dx} \) at the point \( (x, y) \) for each function below.

   a. \( y = \sqrt{3x - 4} \)
   b. \( y = e^{5x} \)
   c. \( y = \ln(5x + 6) \)
   d. \( y = x^4 - 5x + 1 - \frac{1}{x} \)
   e. \( y = x^2 e^{4x} \)
   f. \( y = \cos(4x) \)
   g. \( y = \frac{x}{\sin x} \)
   h. \( y = \frac{2x}{3x - 1} \)

42. Which is a line which can be used to approximate values of \( y = \sqrt[3]{x} \) near \( x = 8 \) ?

   A. \( x - 3y = 2 \)
   B. \( x + 3y = 14 \)
   C. \( x + 12y = 32 \)
   D. \( x - 12y = 8 \)
   E. \( x - 12y = -16 \)
Use this graph of $y = f(x)$ to answer questions #43-45. **Increments on both axes are one.**

43. Over the interval $[-2, 2]$, how many values on $f$ appear to satisfy the conclusion of the Mean Value Theorem?

A. 0  B. 1  C. 2  D. 3  E. 4

44. What is the average rate of change of $f$ over the interval $[-2, 2]$?

A. 1  B. $\frac{3}{2}$  C. 2  D. $\frac{5}{2}$  E. 0

45. Consider the interval $[-2, k]$. For what value of $k$ is the average rate of change of $f$ equal to 0?

46. Find the value of $\frac{d^2y}{dx^2}$ for each function below, at the value $x = c$ given.

a. $y = (4 - x^2)^3$ at $x = 1$

b. $y = \cos(3x)$ at $x = \frac{\pi}{3}$

c. $y = e^{2x^2}$ at $x = 1$

47. A local minimum value of the function $y = \frac{e^x}{x}$ is at $x =$ ?
48. If \( f(x) = \sqrt{2x} \), then \( f'(2) = \)

A. \( \frac{1}{4} \)  
B. \( \frac{1}{2} \)  
C. \( \frac{\sqrt{2}}{2} \)  
D. 1  
E. \( \sqrt{2} \)

49. Find the value(s) of \( c \) that satisfy the conclusion of the Mean Value Theorem for Derivatives for \( f(x) = \sqrt{x-1} \) on \([2,5]\).

50. Find the global extrema of the function \( f(x) = x^2 - 10x + 5 \) on the interval \([-3,8]\).

51. Oil is leaking out of a tank into a reservoir. The rate of flow is measured every two hours for a 12-hour period. The data is in the table below.

<table>
<thead>
<tr>
<th>Time (hr)</th>
<th>Amt (gal/hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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</tr>
<tr>
<td>2</td>
<td>38</td>
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<td>10</td>
<td>18</td>
</tr>
<tr>
<td>12</td>
<td>8</td>
</tr>
</tbody>
</table>

a. Use a left-hand Riemann sum to approximate the area under the rate of flow graph \( f \) where \( f \) is the function whose values are given in the table. (3 equal subdivisions)

b. Use a right-hand Riemann sum to approximate the area under the rate of flow graph \( f \) where \( f \) is the function whose values are given in the table. (3 equal subdivisions)

52. Use a right hand Riemann sum with 4 equal subdivisions to approximate the area bounded by the curve \( y = \frac{1}{x} \) and the x-axis over the interval \([0.5, 2.5]\).

53. A position function \( p(t) = 2t^3 - 15t^2 + 24t + 1 \) gives the position of a particle moving left and right along the x-axis, for \( t \) seconds. Answer each of the following:

a. What is the initial position of the particle?

b. In what direction is the particle moving at \( t = 2 \) seconds?

c. For what interval(s) for \( t \geq 0 \) is the particle moving to the left?

d. What is the acceleration of the particle at \( t=2 \) seconds?